What Is Algebra?

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Let's imagine for a moment that you have a little brother or sister who just started school. Your sibling comes to you with a question: "I'm supposed to learn this stuff called arithmetic. What is that about?" And you, being a helpful older brother or sister, think for a moment and answer something like:

> **Arithmetic is about numbers,**
> **and operations on numbers.**

The next question is "What's an operation?" You might answer "An operation is a special kind of change you can make with numbers." Your sibling then asks "Why would I want to do that?" And you answer "Because there are many different kinds of things in the world, and it's easier to understand how some of them work if you can do that."

But what if someone came to you and asked "I'm supposed to learn this stuff called algebra. What is that about?"

You might be surprised how many people don't have a good answer to that question, even if they have already taken and passed an algebra class. Some might say "Algebra is about solving for x." And that's true, but it isn't enough. Here is a better answer:

> **Algebra is about relationships between numbers,**
> **and operations on relationships.**

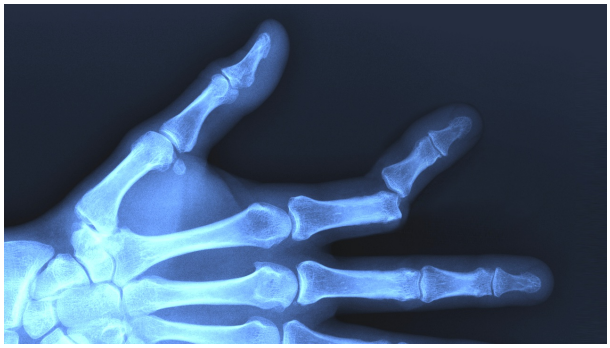
An operation on a relationship is a special kind of change you can make. ***It changes the structure of the relationship, but doesn't change its meaning.*** Why would you want to do that? Because there are many different kinds of things in the world, and it's easier to understand how more of them work if you can do that.

Algebra is kind of like Transformer™ toys. Let's imagine for a moment that Transformers are real. These machines have lots of parts. But unlike most machines, their parts can be rearranged. In one arrangement of parts, it's a truck. In another arrangements of parts, it becomes an airplane. Move the parts around again, and it might turn into a submarine.

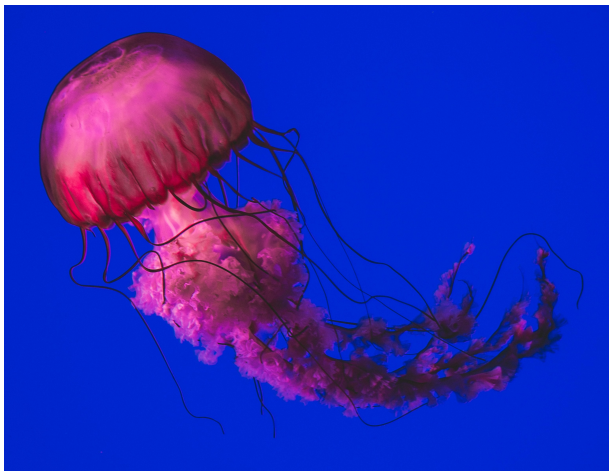
Here is an important question: Why does a Transformer want to transform? And here is an answer: *Because it wants to do something specific, and it can do that more easily in another form.*

Here is another thing that is so obvious, you might not think to notice it, but it is really important: *Even after changing itself around, it is always still the same machine.*

Here is a Transformer that is not imaginary. You don't have to go to the movies to see them. Just like imaginary Transformers, these can be rearranged into many useful forms, depending on what you want them to do.



But you can't move a Transformer's parts around just any way you like. You have to follow particular pathways of change. If you try to ignore this, the machine will break. That hurts. And it can't do much that's useful when broken.



Also, you can't turn a Transformer into just anything at all. There is probably an upper limit on how many things a single Transformer can become. And there are some things it can't become, no matter how much you might want that.

There are many different kinds of relationships between numbers. And people have learned these relationships can be transformed in particular ways, just like Transformer toys. A transformed relationship still *means the same thing*, but it looks different, and it can *do* different things.

When transforming algebraic relationships, we often don't make just one transformation. We might make lots of them, one after another. After only a few changes, it can be hard to remember everything that has happened. So people invented ways of *writing* relationships to help us remember what we have done. An algebraic relationship is an idea that *means* something, and a written algebraic expression is a way of *looking at it* outside of our imagination.

There are lots of ways algebra *can* be written. Over the last few hundred years, people have settled on only one way of writing math that *everyone* is expected to know. Then we can easily communicate math ideas with each other.

But sometimes it is more helpful to write math in other ways. Sometimes we draw pictures. And sometimes we use the familiar math symbols, but arrange them differently to emphasize different things.

Algebra is about transforming relationships. Eventually, people who did this kind of thinking started to notice that it is often easier to transform the *symbols we write* than it is to transform the *ideas in our heads*. They are supposed to *mean* the same thing, right?

This is both a good thing and a bad thing.

It is a good thing because symbols on paper help us remember all the details, and transforming symbols according to known rules can be very efficient. Solving a problem with symbols is like driving a car that gets great gas mileage.

It is a bad thing because sometimes people forget that the symbols mean something. Also, while the symbols are supposed to mean the same thing as the ideas, they aren't always quite as good as the ideas. Even a car that gets great gas mileage can't drive just anywhere, and sometimes it even breaks down.

Let's look more closely at these troublesome aspects.

Imagine you are given a TV remote control, and you are supposed to learn how to use it. Maybe you will be tested on your ability to press all those buttons. But perhaps someone didn't tell you, or perhaps you forgot, that there is a TV across the room, and pressing buttons on the remote *does things* to that TV. What a silly thing to think! But many algebra students are in exactly this situation. They see the symbols on paper, and they try to learn to transform the symbols according to some rules so they can pass a test at school, but they forget that the symbols and transformations *mean something*. You might learn to push all the buttons on your remote, but without the TV, why bother? Without the meaning, it's a pointless game that isn't even fun.

If you master the symbols and the rules for transforming them, you can do a lot of algebra. But you must always be able to look at the symbols and understand what they *mean*. That's because the symbols and their meaning are *related*, but they are *not the same thing*. The symbols will usually help you to a solution of the problem you are working on. But sometimes, they are not enough. Then you have to think about what the symbols mean to figure out what to do next.

And when you have solved a problem using symbols, the meaning you find at the end is still the reason you did the work.

Learning algebra means these five things:

1) **Understanding relationships between numbers.**
2) **Using symbols to write those relationships.**
3) **Understanding transformations of relationships that change their structure without changing their meaning.**
4) **Using rules for transforming symbolic relationships that change their structure without changing their meaning.**
5) **Understanding the world around you in terms of numbers, relationships, and transformations.**

Three of these are the things that really matter.

Two of these are tools we use to help us do the things that matter.